Engineering Notes

Measurement-Based Modeling Methodology for Ballistic Missile Trajectory Simulation

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I. Introduction

IN THE field of ballistic missiles, the problem of tracking the missile until the impact point is not only an important problem but also quite complex. The problem is made further complex by the approximations that are usually required to be introduced in the mathematical models, which are employed for predicting a reference trajectory for tracking. In addition, there are situations where an existing missile system needs to be redesigned/redeployed for a different mission scenario and thus needs tuning/modification of the trajectory simulation models. The present Note proposes a measurement-based model-updating strategy to create, as well as modify, the simulation models.

In literature, there have been many studies [1–4] that have investigated the problem of creating a trajectory model using different philosophies and strategies. These include a conditional boost-phase trajectory-estimation method based on a ballistic missile information database and classification under an uncertain control environment, missile tracking using a priori information (e.g., launch position, launch time, burnout time, initial mass, and fuel burn rate), and posing of the trajectory estimation as a nonlinear parameter estimation problem. The previous studies have also shown that good trajectory estimates are possible with nonlinear filtering techniques [5–8]; for example, the unscented Kalman filter, the interactive multiple model filter, and the extended Kalman filter (EKF). Among these techniques, the EKF-based techniques are the most widely used in nonlinear filtering algorithms for state estimation, including target tracking. However, none of the studies have addressed the issue of model building in the case when the missile data set is incomplete. Therefore, the aim of the present study is to evolve a trajectory-measurement-based algorithm for arriving at a consistent and robust mathematical model for predicting the trajectories of a typical ballistic missile, for which some aspect of its description is either missing or known with poor accuracy. The proposed methodology makes use of a standard nonlinear dynamic model structure, along with the EKF algorithm, to establish a model evolution procedure. A typical ballistic missile configuration, along with outputs commonly measured during a realistic tracking scenario, is used to verify the feasibility of the proposed modeling strategy.

II. Ballistic Missile Flight Dynamic Model Structure

The ballistic missile covers long distances and has atmospheric and exoatmospheric phases in the trajectory; therefore, the dynamic model needs to incorporate all the contributions. The model structure proposed in the present study has taken note of this requirement and has adapted the nonlinear dynamic model proposed by Tewari [4]. Figure 1 gives the Earth-centric inertial and the local horizon-based coordinate axes systems for expressing the general nonlinear dynamical equations of motion of the ballistic missile.

It is mentioned here that the centripetal and Coriolis accelerations due to the rotation of the Earth, along with the thrust, aerodynamic drag, and gravity forces, are included in the proposed flight dynamic model. The kinematic and dynamic equations of motion relative to a rotating Earth are given next, as described in [4]:

$$\dot{r} = v\sin\phi; \qquad \dot{\delta} = \frac{v}{r}\cos\phi\cos A; \qquad \dot{\lambda} = \frac{v\cos\phi\sin A}{r\cos\delta} \tag{1}$$

$$\dot{v} = \frac{f_T}{m} - \frac{\rho v^2 S C_d}{2m} - g_r\sin\phi + g_n\cos\phi\cos A - r\omega^2\cos\delta(\cos\phi\cos A\sin\delta - \sin\phi\cos\delta) \tag{2}$$

$$\dot{A} = \frac{v}{r}\cos\phi\sin A\tan\delta - \frac{g_n\sin A}{v\cos\phi} + \frac{r\omega^2\sin A\sin\delta\cos\delta}{v\cos\phi} + 2\omega(\sin\delta - \cos A\cos\delta\tan\phi) \tag{3}$$

$$\dot{\phi} = \frac{v}{r}\cos\phi - \frac{g_r}{v}\cos\phi - \frac{g_n}{v}\sin\phi\cos A + \frac{r\omega^2}{v}\cos\delta(\sin\phi\cos A\sin\delta + \cos\phi\cos\delta) + 2\omega\sin A\cos\delta \tag{4}$$

The preceding coupled nonlinear differential equations cannot be solved in a closed form; hence, an iterative numerical procedure in the form of a fourth-order Runge–Kutta integration algorithm is used in the present study.

III. Model Building and Parameter Estimation Strategy

In the present study, it is a priori assumed that the mass properties and propulsion parameters are usually known quantities from the overall missile classification. It is further assumed that, for an unknown mission flown by a missile with known classification, both the drag profile and the boost-phase pitch program are unknown, with the actual trajectory being known only through explicit measurement of the applicable parameters. In view of the fact that both of these quantities are required for generating the flown trajectory using a simulation model, we have proposed a model-building methodology that arrives at the estimates of the drag profile as well as the pitch and heading angle variations from a measured trajectory. The drag coefficient is estimated from the reference trajectory data using the following algorithm. The missile speed and altitude information from the trajectory is used to estimate the flight Mach number as a function of time, using the standard atmosphere (i.e., exponential density model). From this information, a drag coefficient curve can be hypothesized as a function of Mach number in the following manner:

$$C_d = \frac{2}{\rho v^2(k)S}\left(f_T - m\left\{\dot{v}(k) + \frac{\dot{r}(k)}{v(k)}[g_r - r(k)\omega^2\cos^2\delta(k)] + \frac{\dot{\delta}(k)r(k)}{v(k)}[r(k)\omega^2\cos\delta(k)\sin\delta(k) - g_n]\right\}\right) \tag{5}$$

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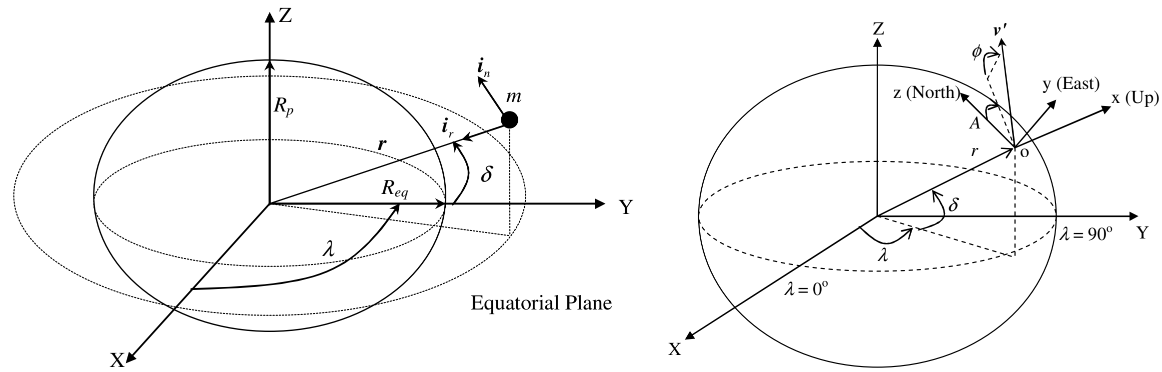

Fig. 1 Earth-centered and local horizon-based coordinate frames.

$$\dot{\delta}(k) = \{\delta[(k+1)T_s] - \delta(kT_s)\}/T_s$$
$$\dot{r}(k) = \{r[(k+1)T_s] - r(kT_s)\}/T_s \tag{6}$$

and

$$\dot{v}(k) = \{v[(k+1)T_s] - v(kT_s)\}/T_s$$

It should be mentioned here that the drag estimation algorithm is applicable only to trajectories below 50 km altitude, as the drag is quite small at higher altitudes. An estimate of the constant thrust force is obtained from the missile classification. It is further noted that the boost phase is assumed with a constant thrust. In cases where the thrust varies during the boost phase, an average constant value can be taken as the thrust profile. The flight-path and heading angles are estimated from a measured trajectory using the following algorithm:

$$\phi_e(k) = \sin^{-1}\left(\frac{\dot{r}(k)}{v(k)}\right)$$
$$A_e(k) = \begin{cases} \tan^{-1}[\dot{\lambda}(k)\cos\delta(k)/\dot{\delta}(k)] & \text{if } \dot{\delta}(k) > 0 \\ \pi + \tan^{-1}[\dot{\lambda}(k)\cos\delta(k)/\dot{\delta}(k)] & \text{if } \dot{\delta}(k) < 0 \end{cases}$$
$$\dot{\lambda}(k) = \{\lambda[(k+1)T_s] - \lambda(kT_s)\}/T_s \tag{7}$$

IV. Extended Kalman Filter Realization

In the present Note, the basic EKF formulation applicable for online trajectory tracking is presented along with the necessary equations for its implementation in the following equations:

$$F = \left.\frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}}\right|_{x=\hat{x}} = \begin{bmatrix} \frac{\partial\dot{\delta}}{\partial\delta} & \frac{\partial\dot{\lambda}}{\partial\delta} & \frac{\partial\dot{r}}{\partial\delta} & \frac{\partial\dot{v}}{\partial\delta} & \frac{\partial\dot{\phi}}{\partial\delta} & \frac{\partial\dot{A}}{\partial\delta} \\ \frac{\partial\dot{\delta}}{\partial\lambda} & \frac{\partial\dot{\lambda}}{\partial\lambda} & \frac{\partial\dot{r}}{\partial\lambda} & \frac{\partial\dot{v}}{\partial\lambda} & \frac{\partial\dot{\phi}}{\partial\lambda} & \frac{\partial\dot{A}}{\partial\lambda} \\ \frac{\partial\dot{\delta}}{\partial r} & \frac{\partial\dot{\lambda}}{\partial r} & \frac{\partial\dot{r}}{\partial r} & \frac{\partial\dot{v}}{\partial r} & \frac{\partial\dot{\phi}}{\partial r} & \frac{\partial\dot{A}}{\partial r} \\ \frac{\partial\dot{\delta}}{\partial v} & \frac{\partial\dot{\lambda}}{\partial v} & \frac{\partial\dot{r}}{\partial v} & \frac{\partial\dot{v}}{\partial v} & \frac{\partial\dot{\phi}}{\partial v} & \frac{\partial\dot{A}}{\partial v} \\ \frac{\partial\dot{\delta}}{\partial\phi} & \frac{\partial\dot{\lambda}}{\partial\phi} & \frac{\partial\dot{r}}{\partial\phi} & \frac{\partial\dot{v}}{\partial\phi} & \frac{\partial\dot{\phi}}{\partial\phi} & \frac{\partial\dot{A}}{\partial\phi} \\ \frac{\partial\dot{\delta}}{\partial A} & \frac{\partial\dot{\lambda}}{\partial A} & \frac{\partial\dot{r}}{\partial A} & \frac{\partial\dot{v}}{\partial A} & \frac{\partial\dot{\phi}}{\partial A} & \frac{\partial\dot{A}}{\partial A} \end{bmatrix}_{x=\hat{x}}$$

$$H = \left.\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}}\right|_{x=\hat{x}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

The tracking is done in a transformed coordinate system, where we assume that the measurements are available as $\delta(k)$, $\lambda(k)$, $r(k)$, and $v(k)$. It is seen from the preceding equation that pseudomeasurements of flight-path and heading angles are also included to improve the tracking of the respective states. The discrete Riccati equation, along with its implementation details, is as follows [9]:

$$\boldsymbol{\Phi}_k = \boldsymbol{I} + \boldsymbol{F}T_s; \qquad \boldsymbol{Q}_k = \int_0^{T_s} \boldsymbol{\Phi}(\tau)\boldsymbol{Q}\boldsymbol{\Phi}^T(\tau)\,\mathrm{d}\tau$$
$$\boldsymbol{M}_k = \boldsymbol{\Phi}_k\boldsymbol{P}_{k-1}\boldsymbol{\Phi}_k^T + \boldsymbol{Q}_k \tag{9}$$

$$\boldsymbol{K}_k = \boldsymbol{M}_k\boldsymbol{H}^T(\boldsymbol{H}\boldsymbol{M}_kH^T + \boldsymbol{R}_k)^{-1}; \qquad \boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H})\boldsymbol{M}_k \tag{10}$$

The updated state estimate is obtained by

$$\hat{\boldsymbol{x}}_k = \bar{\boldsymbol{x}}_k + \boldsymbol{K}_k[z_k - h(\bar{\boldsymbol{x}}_k)]; \qquad \bar{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k-1} + \hat{\dot{\boldsymbol{x}}}_{k-1}\boldsymbol{T}_s$$
$$\hat{\dot{\boldsymbol{x}}}_{k-1} = f(\hat{\boldsymbol{x}}_{k-1}) \tag{11}$$

The state estimate $\bar{x}_k$ is propagated forward by integrating the actual nonlinear differential equations at every sampling interval. It is seen from Eq. (11) that the first-order Euler integration is used to propagate the state estimate, which is considered to be adequate in the present case.

V. Model Validation Through Simulation Results

To establish the validity and performance of the proposed simulation model, for which the structure is given by Eqs. (1–4) and uses the estimates of the drag profile as well as flight-path and heading angles, a representative single-stage ballistic missile is chosen, as shown in Fig. 2 and Table 1. To verify the applicability of the proposed model-building strategy, the following methodology is used. First, a forward solution of the trajectory simulation is carried out for the preceding set of parameters using a standalone trajectory simulation software [10], with the initial launch conditions given in Table 2. The preceding software is used in a black-box mode, which also has many elements necessary for generating the trajectory so that the trajectory output from the preceding software is a result of many more parameters than those listed in Table 1. The trajectory generated is sampled at every 1 s, which then serves as a reference (or measured) trajectory and is treated as the actual dynamics of the sample missile. Next, the so-called measured trajectory is given to the estimation algorithm developed in this Note, which estimates the

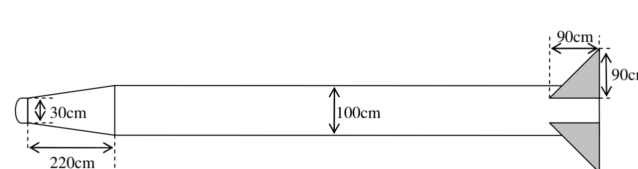

Fig. 2 Geometric configuration of a typical single-stage ballistic missile.

Table 1 Ballistic missile configuration

Parameter	Value
Number of stages	1
Gross mass, kg	19,900
Fuel mass, kg	16,000
Burn time, s	70
Specific impulse (I_{SP}, s	240
Stage diameter, m	0.88
Payload mass, kg	1000
Warhead diameter, m	0.84

Table 2 Initial launch conditions

Parameter	Value
Launch site latitude, deg	40.85
Launch site longitude, deg	129.67
Launch azimuth, deg	40

actual drag profile and the actual pitch program as unknown quantities.

A. Drag Profile Estimation and Verification

To establish the accuracy of the estimated drag profile, the same profile is generated from both the simulation tool [10] as well as from the empirical relations for the drag coefficient given by Missile DATCOM [11]. Figure 3 gives a comparison of the drag profile obtained as a function of the Mach number, from the three different methods, by assuming angle of attack to be zero, which is normally the case for ballistic missiles. In the present case, the constant thrust in boost phase is separately estimated as 5.38×10^5 N for estimating the drag coefficient. However, during the reentry phase, the estimation algorithm sets the thrust to zero, while the total vehicle mass is taken to be the mass of only the payload. It is noted here that the DATCOM predictions are known to be more accurate in the Mach number range from 1.5 to 2.5 [12], which is also seen clearly in Fig. 3. It is also clear from Fig. 3 that the estimated drag profile is within 10% of the reference trajectory drag profile, bringing out the overall acceptability of the drag profile estimation algorithm. It is also seen that outside of the Mach number range of 1.5–2.5, DATCOM predictions show large deviations from both estimated as well as actual drag profiles.

B. Flight-Path and Heading Angle Estimations

The flight-path and heading angles are estimated from the reference sampled trajectory using Eq. (7), and the corresponding estimation results are given in Figs. 4 and 5, respectively. It may be noted here that the actual trajectory solutions, in terms of the latitude, longitude, altitude, and velocity profiles, are corrupted with a zero-mean random measurement noise, having standard deviations of 0.01°, 0.01°, 100 m, and 100 m/s, respectively It can be seen from the variation of flight-path angle that there are large variations for up to 70 s of flight time, which reduce to a monotonically decreasing trend beyond 70 s. This can be clearly attributed to the presence of thrust and a specific pitch program, and it can be said that flight-path angle variations during this phase give a reasonably good estimate of the actual pitch program that the missile is executing. It can also be seen that. although the estimates are corrupted with the noise, the mean follows the expected trend. Figure 5 shows that the estimated heading angle varies within a small range of 10°: i.e., from 40 to 50°.

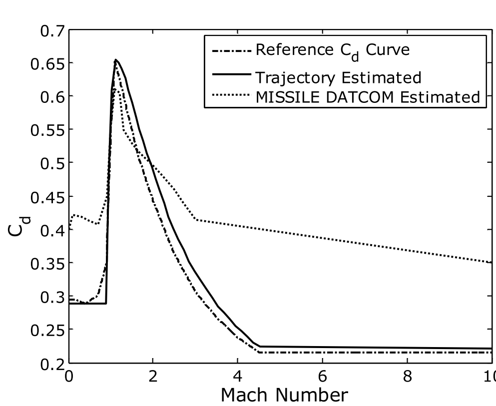

Fig. 3 Comparison of the variation of the drag coefficient.

C. Verification of Estimated Parameters Through Trajectory Reconstruction

To further verify the estimated drag profile as well as the flight-path and heading angle variations, the reference trajectory is reconstructed using the estimated parameters for the same missile configuration and initial conditions. Table 3 gives a comparison of the actual trajectory parameters, with the parameters predicted using the estimated profiles, and it is seen that the predicted values are fairly close to the actual values.

Figures 6–9 present comparisons of the variations in the latitude, longitude, altitude, and velocity profiles.

It is observed from these plots that the trajectory predicted from estimated parameters matches reasonably well with the actual trajectory, even though the pitch program used is only an estimate and not the actual one. In addition, it is seen that the time of flight, ground range, and the apogee altitude of the simulated trajectory are all within 3.5% of their actual values; therefore, it can be concluded

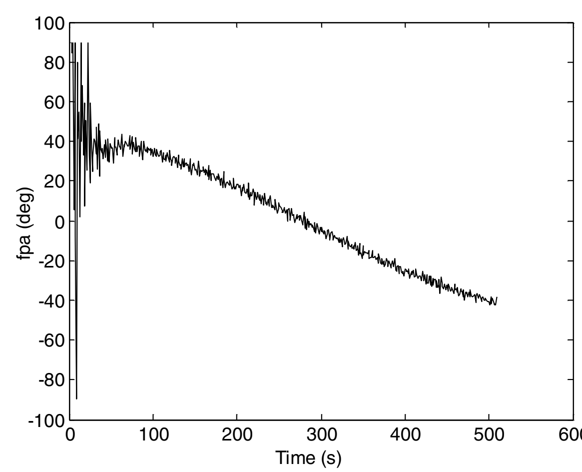

Fig. 4 Flight-path angle variation.

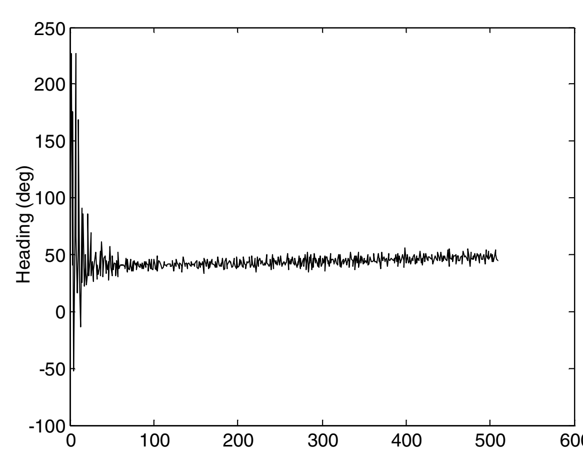

Fig. 5 Heading angle variation.

Table 3 Simulation outputs

	Actual	Model output
Time of flight, s	509	519
Range, km	978.28	1012.00
Apogee height, km	223.7	231.5
Azimuth, deg	41.05	40.73

Table 4 Simulation outputs

	Actual	Model output
Time of flight, s	525.90	530.60
Range, km	1085.79	1110.17
Apogee height, km	247.7	246.2
Azimuth, deg	261.25	261.28

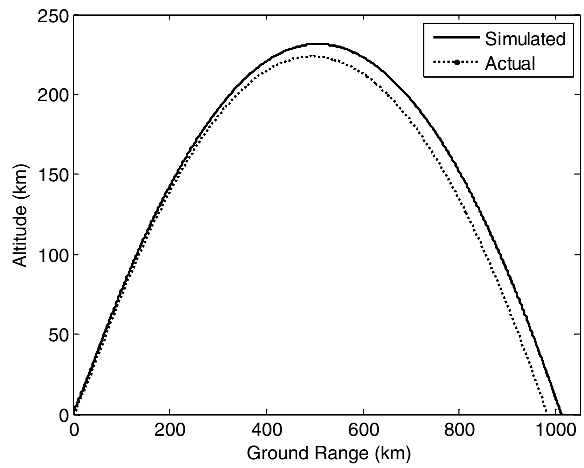

Fig. 6 Altitude vs ground range.

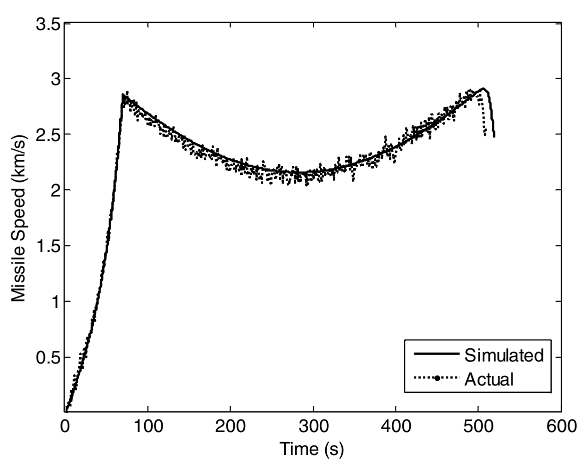

Fig. 7 Missile speed variation.

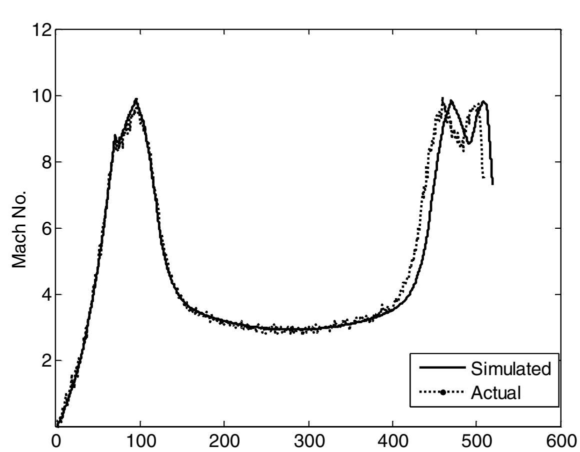

Fig. 8 Mach number variation.

that use of the estimated parameters in the proposed nonlinear dynamical model structure, which are based on the actual trajectory measurements, not only results in a more realistic simulation model but also provides a good approximation to the actual trajectory. The proposed methodology is also able to provide a fairly good estimate of the actual pitch program; thus, it is a distinct advantage in cases where information about such pitch programs is not readily available.

D. Efficacy of Estimated Profiles in Predicting Trajectories for Different Missions

To establish the usefulness of the developed model-building strategy for creating generic and robust mathematical models capable of providing acceptable trajectory predictions for new missions for the same missile, additional reference trajectories are predicted for a separate mission by varying the payload mass, the launch point, and the launch azimuth. In these cases, both estimated pitch program and the pitch program of the reference trajectory, along with the missile configuration used, are kept the same, except that the payload mass is changed to 500 kg. Table 4 shows the comparison between the actual trajectory created using the simulation tool and the output from the proposed mathematical model, which is based on the estimated parameters. It can be seen that, although the estimated parameters are for a different mission, they are able to provide a reasonably accurate prediction of the missile dynamic behavior for an entirely different mission. Figures 10 and 11 show the trajectory profile comparison, and it is seen that the actual trajectory for this mission is fairly close to the predicted trajectory based on the estimates for the same missile. From these results, we can see that the simulation model produces satisfactory results, even for significantly different launch conditions.

Finally, the proposed missile-modeling strategy, based on the estimated parameters, can also be used to predict the entire ballistic trajectory by using only a small tracked portion of the total trajectory. For the test case considered in the present study, a portion of the reference trajectory just before the impact is taken, and the initial conditions for the required reverse integration are derived from this portion of the reference trajectory. The reference trajectory generated for the same missile configuration and initial conditions, as given in Tables 1 and 2, respectively, is used, and it is assumed that the trajectory data are free from any measurement noise. The reverse

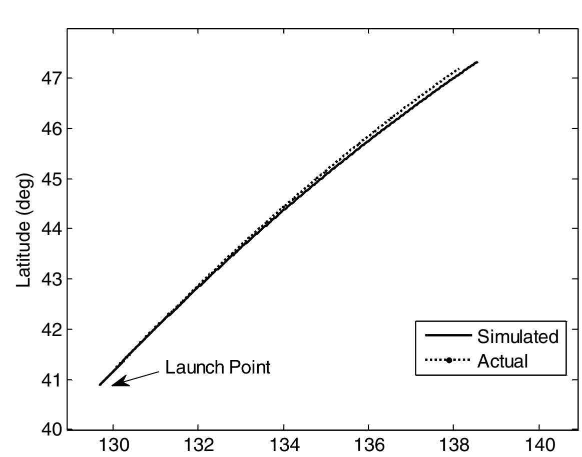

Fig. 9 Trajectory ground trace.

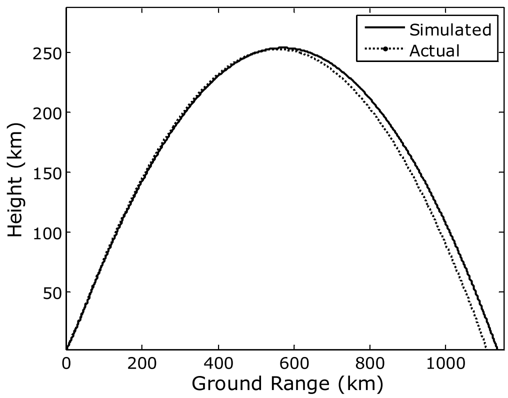

Fig. 10 Altitude vs ground range.

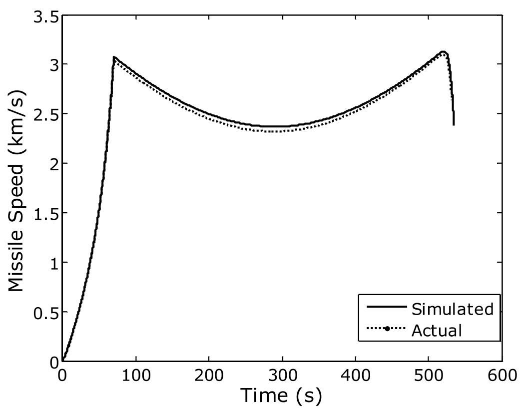

Fig. 11 Missile speed variation.

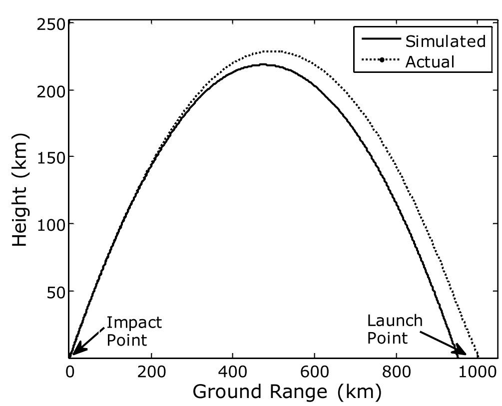

Fig. 12 Altitude vs ground range.

integration is used to reproduce the entire ballistic trajectory, and the launch point is predicted by extending this ballistic trajectory until the impact point on the Earth. The thrust is considered to be zero throughout the trajectory, and the mass is the same as that of the payload. It is seen from Figs. 12 and 13 that the predicted launch point is about 49 km short of the actual launch point (i.e., an error of about 5% in the prediction), while it is seen that the actual launch point of the reference trajectory continues to lie along the same heading as that of the predicted trajectory. It is further seen that the velocity profile is reproduced faithfully until the thrust cutoff point, after which it starts diverging. These deviations can be attributed to the unmodeled dynamics during the boost phase, although the predicted ground trace matches reasonably well with that of the actual trajectory.

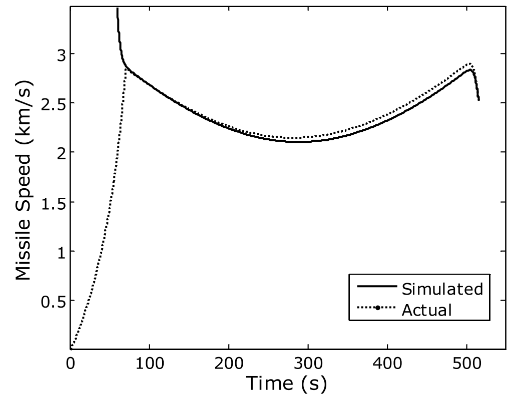

Fig. 13 Missile speed variation.

V. Conclusions

A new modeling methodology, based on the model parameters estimated from a measured trajectory, is proposed for ballistic missiles having an incomplete data set for creating generic simulation models. Starting from a general nonlinear dynamic model structure for the missile, an algorithm is evolved for estimating the drag profile, along with the boost-phase pitch program, using the EKF formulation. The dynamic model is then synthesized with the estimated parameters to create a complete mathematical model capable of predicting ballistic missile trajectories for a large class of missions. Typical ballistic missile geometry is used to demonstrate the applicability and usefulness of the proposed model-building strategy, and it is shown that the estimated model reproduces the actual dynamics of the ballistic missile throughout the trajectory with a reasonable degree of accuracy. Thus, the present Note establishes a new methodology for dynamic model building, using measurement-based estimates for the trajectory parameters, for the ballistic missiles that have an incomplete data set.

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